Graphy Theory Week 12

Directed Graph

Theorem:

A diagraph D contains a directed graph of length $\chi - 1$

Theorem: (Chvátal. Lavász, 1974)

In a directed graph G, there is always an independent set of vertices such that given any $v \notin S$, there is an $u \in S$ with $d(u,v) \leq 2$

Proof:

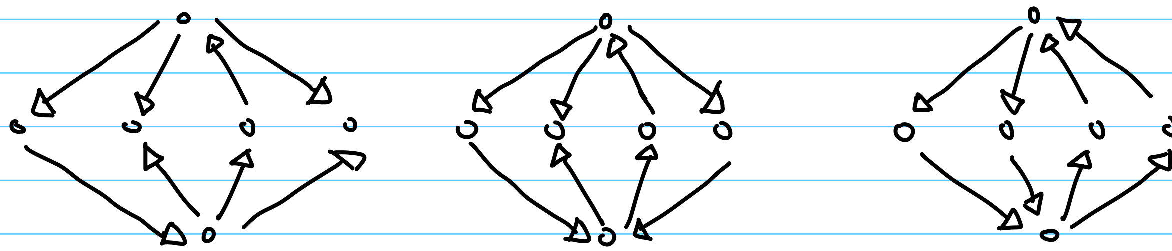
By induction. Let $w \in G$; Let G' be the subgraph of G induced by $\{u \mid d(\overrightarrow{w,u}) \geq 2\}$. $\therefore \exists S' \subseteq G'$

If $d(\overrightarrow{u,w}) \leq 1$ for some $u \in S'$, Let $S = S'$

Otherwise we set $S = S' \cup \{w\}$ $\square$

Connectivity

Example:



Weak Conn. Single Conn Strong Conn.

$P(u,v)$ or $P(v,u)$ $P(u,v)$ and $P(v,u)$

Theorem:

$\vec{G}$ is single connected graph if and only if All vertices in $\vec{G}$ are in one directed path.

Theorem:

$\vec{G}$ is strong connected digraph if and only if any vertices in $\vec{G}$ are in one directed closed path.

Theorem (Robbins, 1939):

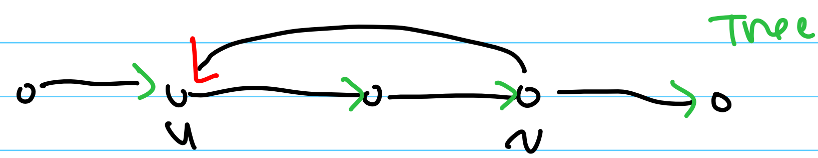
Undirected graph G can be orientated to strong connected digraph if and only if G is connected and no cut edge (G is 2-edge connected)

→ Hopcroft-Tarjan Orientation Algo.

<1> Find directed Tree T

(2) For every $e = uv$ not in Tree,

Give an orientation



Euler and Hamilton Digraph

Theorem:

Non-trivial weak connected digraph $\vec{G}$ is Euler digraph if and only if $\forall v \in V(\vec{G})$: $d^+(v) = d^-(v)$

Theorem: (Meyniel, 1973)

$\vec{G}$ is strong connected digraph, for any two dis adjacent u, v, have $d(u) + d(v) \geq 2\nu - 1$, then $\vec{G}$ is

Hamilton Digraph.

Tournament 竞赛图

Definition

An orientation of a complete graph is called a Tournament.

Theorem:

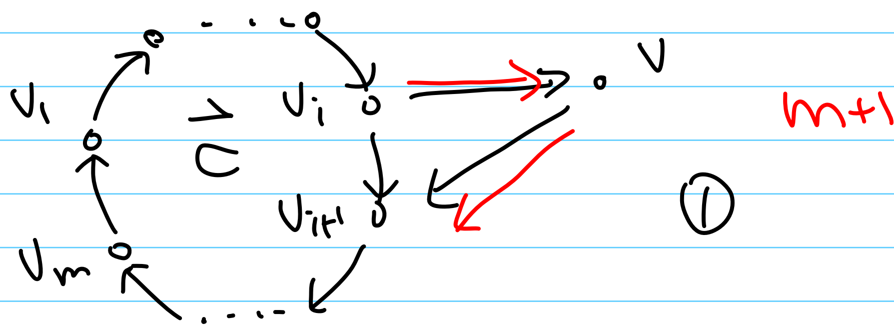
A Tournament contains a vertex from which every other vertex is reachable by a directed path of length at most two.

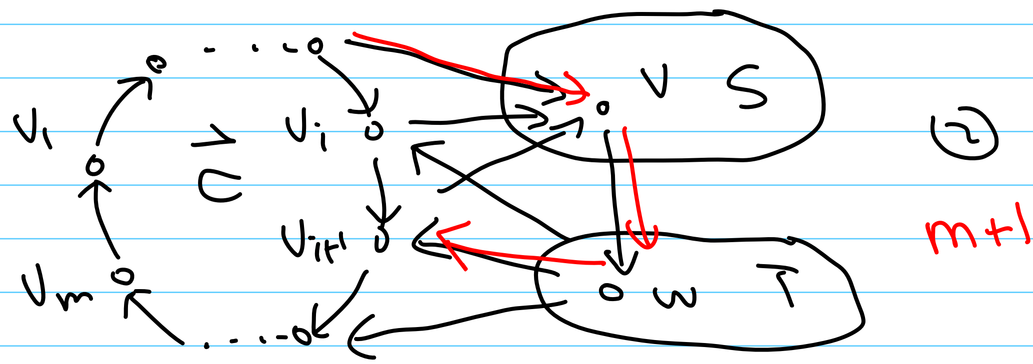
Theorem: (Rédei, 1934)

Every tournament has a directed Hamilton path.

Theorem (Moon, 1966)

Strong Connectivity Tournament with $v \geq 3$. Every vertex is in a directed cycle with length k ($k = 3, 4, \cdots, \nu$)





<u>Corollary</u>: (Camion, 1959)
Strong Connectivity Tournament is Hamilton Directed Graph.