Graphy Theory Week 11 - Week 12

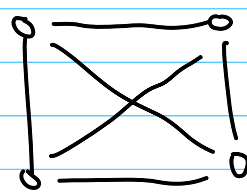
Planar Graphy

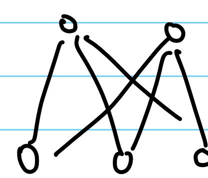
Definition

A graph is said to be embeddable in the plane, or planar, if it can be drawn in the plane so that its edges intersect only at their ends

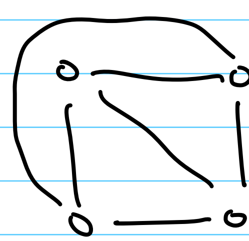
Example

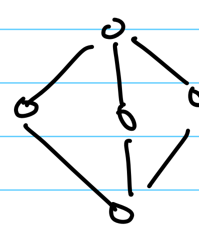
Planar Graph





Planar Embedding





(Plane Graph)

Definition

Such a drawing of a planar graph is called a planar embedding of G. Sometimes refer to a planar embedding of planar graph as Plane Graph.

Faces

Definition

If $G = (V, E)$ is a plane graph, then G divides the plane into connected regions which are called faces. There is always one unbounded face called the Infinite face.

The # of edges of a face's boundary is called the degree of face.

Theorem:

$$\sum_{i=1}^{\varphi} \deg(f_i) = 2\varepsilon(G)$$

Theorem:

For a maximal plane graph at least 3 vertex, Every face's degree must be 3.

Euler's formula

Theorem: (Euler, 1753)

G is Connected plane graph.

$$\nu - \varepsilon + \varphi = 2.$$

↑ 面数.

※ **Theorem:**

G is connected plane graph, $\deg(f) \ge \ell \ge 3$. Then

$$\varepsilon \le \frac{\ell}{\ell-2}(\nu-2)$$

Proof:

$$2\varepsilon = \sum_{i}^{\varphi} d(f_i) \ge \ell \cdot \varphi = \ell(2+\varepsilon-\nu)$$

$$\therefore\ 2\varepsilon \ge \ell(2+\varepsilon-\nu)$$

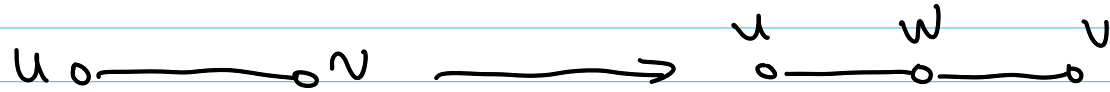
$$\therefore\ 2\varepsilon - \ell\varepsilon \ge \ell(2-\nu)$$

$$\therefore\ \varepsilon(\ell-2) \le \ell(\nu-2)$$

$$\therefore\ \varepsilon \le \frac{\ell}{\ell-2}(\nu-2) \qquad \blacksquare$$

$$\longrightarrow\ \varepsilon \le \frac{\ell}{\ell-2}(\nu-\omega-1) \quad (\omega \ge 1)$$

Edge Subdivision:



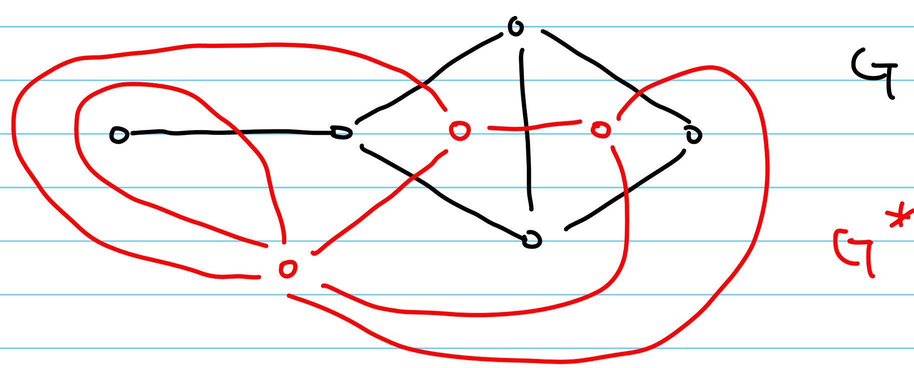
Edge Contraction:

Dual Graph

Definition

Corresponding to each face f of G, there is a vertex f^* of G^*.
two vertices f^* and g^* are joined by edge e^* in G^* iff f and g are seperated by edge e in G

Example:



Theorem

In G^*:

<1> $v^* = \varphi$ <4> $d_{G}(v_i^*) = \deg(f_i)$

<2> $\varepsilon^* = \varepsilon$

<3> $\varphi^* = v$