Chapter 5 Charles

Domination Set

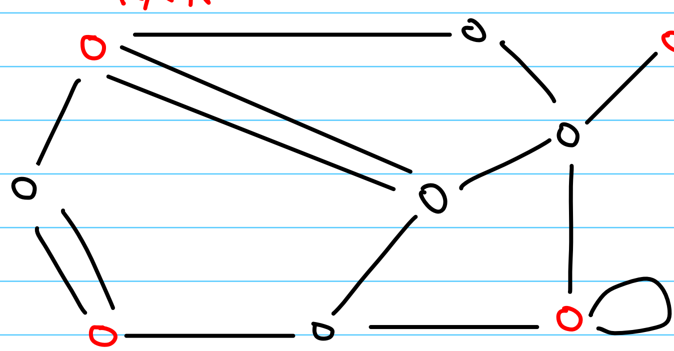
Definition:

A dominating set for a graph is a subset D of its vertices, such that any vertex of G is either in D or has a neighbor in D.

Example: (Minimal VS. Minmum)

Minimal Dominating Set

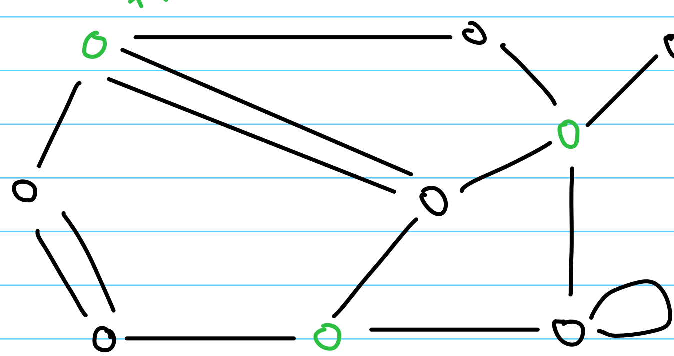
极小



$\forall$ proper subset is not dominating set.

Minimum Dominating Set $|D| = \gamma = 3$

最小



With minmum # of vertices of dominating set.

Theorem:

G has no isolate vertex, D_1 is a minimal dominating set, then $\bar{D} = V(G) - D_1$ is also a dominating set.

* Proof by contradiction

Theorem:

Dominating set D of graph G is a minimal dominating set if and only if every vertex of D satisfies one of following

(1) $N(v) \cap D = \phi$

(2) $\exists u \in V(G) - D : N(u) \cap D = \{v\}$

Theorem:

If G has no isolated vertex, then

$$\gamma(G) \le \frac{\nu}{2}$$

Theorem (Arnautov 1974)

$$\gamma(G) \leq \frac{1+\ln(\delta+1)}{1+\delta}\,\nu$$

Vertex Indpendent Set

Definition

A subset S of V is called an independent set of G if no two vertices of S are adjacent in G.

$\alpha(G)$

$\longrightarrow$ Maximal and maximum Indep.

Theorem.

Maximal independent set of graph G must be minimal dominating set.

Theorem

If I is independent set, then it is maximal independent set if and only if it is dominating set.

Theorem:

For any graph: $\alpha(G) \geq \gamma(G)$

Theorem (Bondy, 1978)

$\nu(G) \geq 2$. If any two non-adjacent vertice x and y have $d_G(x) + d_G(y) \geq \nu(G)$

Then $\alpha(G) \leq \kappa(G)$ (Connectivity)

Theorem (Chvátal & Erdös, 1972)

$|G| = \nu \geq 3$. If $\kappa(G) \geq \alpha(G)$, then G is Hamiltonian.

Vertex Covering Set

Definition:

A subset K of V such that every edge of G has at least one end in K is called a covering of G. The number of vertices in a minimum covering of G is covering number $\beta(G)$

Theorem:

A set $F \subseteq V$ is an independent set of G if and only if $V(G) - F$ is a covering of G.

Corollary:

F is minimal covering $\Leftrightarrow$ $V(G) - F$ is maximal independent set.

$$\alpha(G) + \beta(G) = \nu.$$

Edge Independent

Definition:

Matching M $\leftrightarrow$ Edge Independent

$\alpha'(G)$: edge independent number.

Theorem:

For Any G without loop. $\alpha'(G) \leq \beta(G)$

Theorem (König, Egerváry. 1931):

For Bipartite Graph, $\alpha'(G) = \beta(G)$

Edge Covering.

An edge covering of G is a subset L of E such that each vertex of G is an end of some edge in L.

Edge covering number : $\beta'(G)$ (minimum)

Theorem:

$\delta(G) > 0$, then $\alpha'(G) \leq \beta'(G)$,

$\alpha'(G) = \beta'(G)$ if and only if G has perfect matching.

Theorem (Gallai, 1959):

$\delta(G) > 0$, then $\alpha'(G) + \beta'(G) = \nu$.