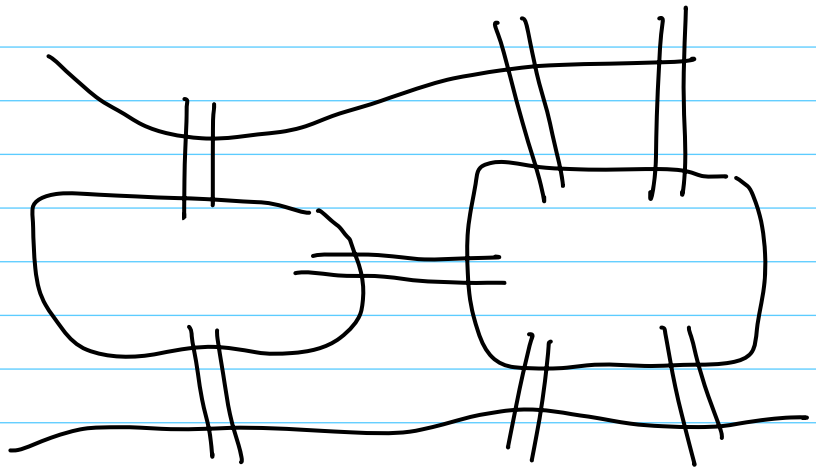
Chapter 4. Charles

Euler Graph

7- Bridge Problem



Definition:

An Euler tour is a tour which traverses each edge exactly once. A graph is Eulerian if it contains an Euler tour.

Theorem (Euler, 1736):

A nonempty connected graph is Eulerian if and only if it has

no vertices of odd degree.

Corollary =

A connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

Theorem

G 可一笔画成当且仅当它最多有2个奇度顶点.

Corollary

G 可k笔画成当且仅当它最多有2k个奇度顶点.

Determine Euler Tour in Eulerian G.

——→ Fluery Algorithm. (1921)

Tracing out a trail. subject to the one condition that a cut edge of the untraced subgraph is taken only if there is no alternatives.

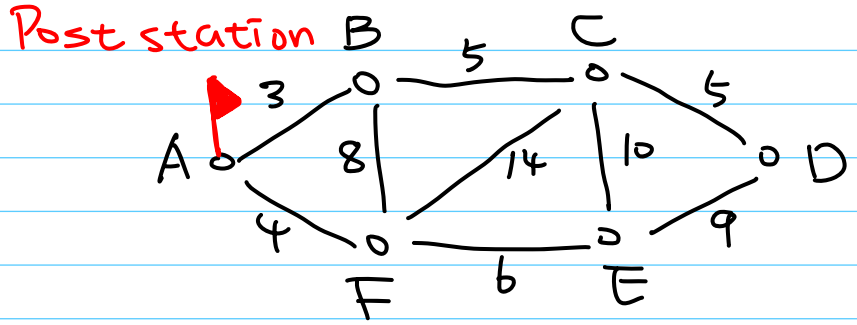
Application: Chinese Postman Problem.

(Prime Problem + Weighted)

一. 混合奇偶点图上作业法

二. Edmonds – Johnson Method

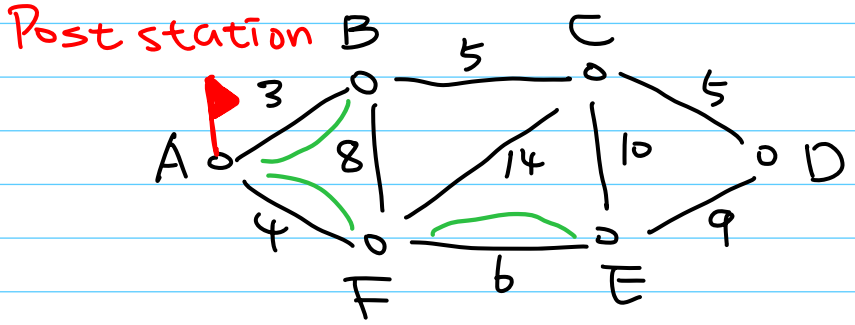
Example:



① Find odd vertices B, E

② Find shortes path (B, E) = BAFE

③ Add Multiedge BA, AF, FE



④ Flewry Algorithm Get tour.

Hamilton Graph

Definition:

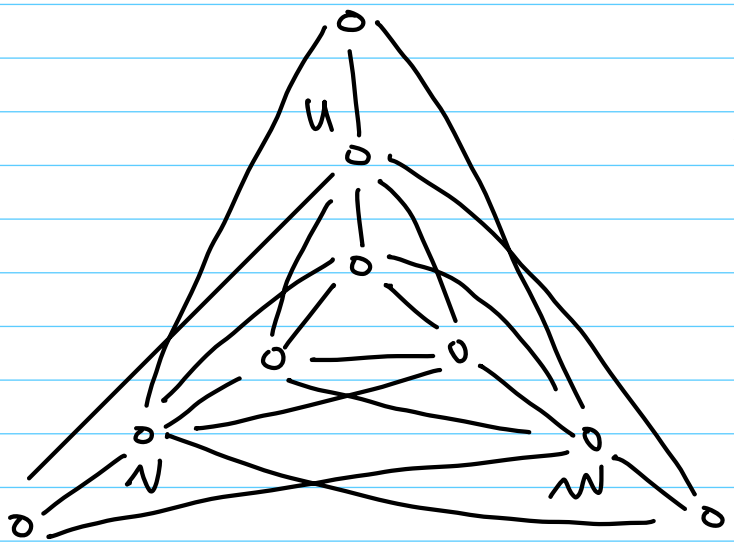
A Hamilton cycle is a cycle in a graph that visits each vertex exactly once. A graph is Hamiltonian if it contains a Hamilton cycle.

Theorem:

If G is a Hamiltonian then, for every nonempty proper subset S of V

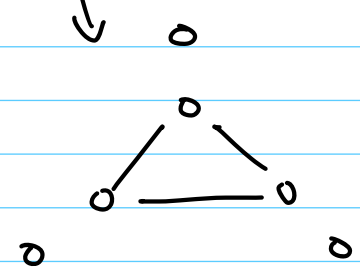
$$w(G-S) \leq |S|$$

Example:



$S = \{u, v, w\}$

$w(G-S) = 4 > \{S\}$

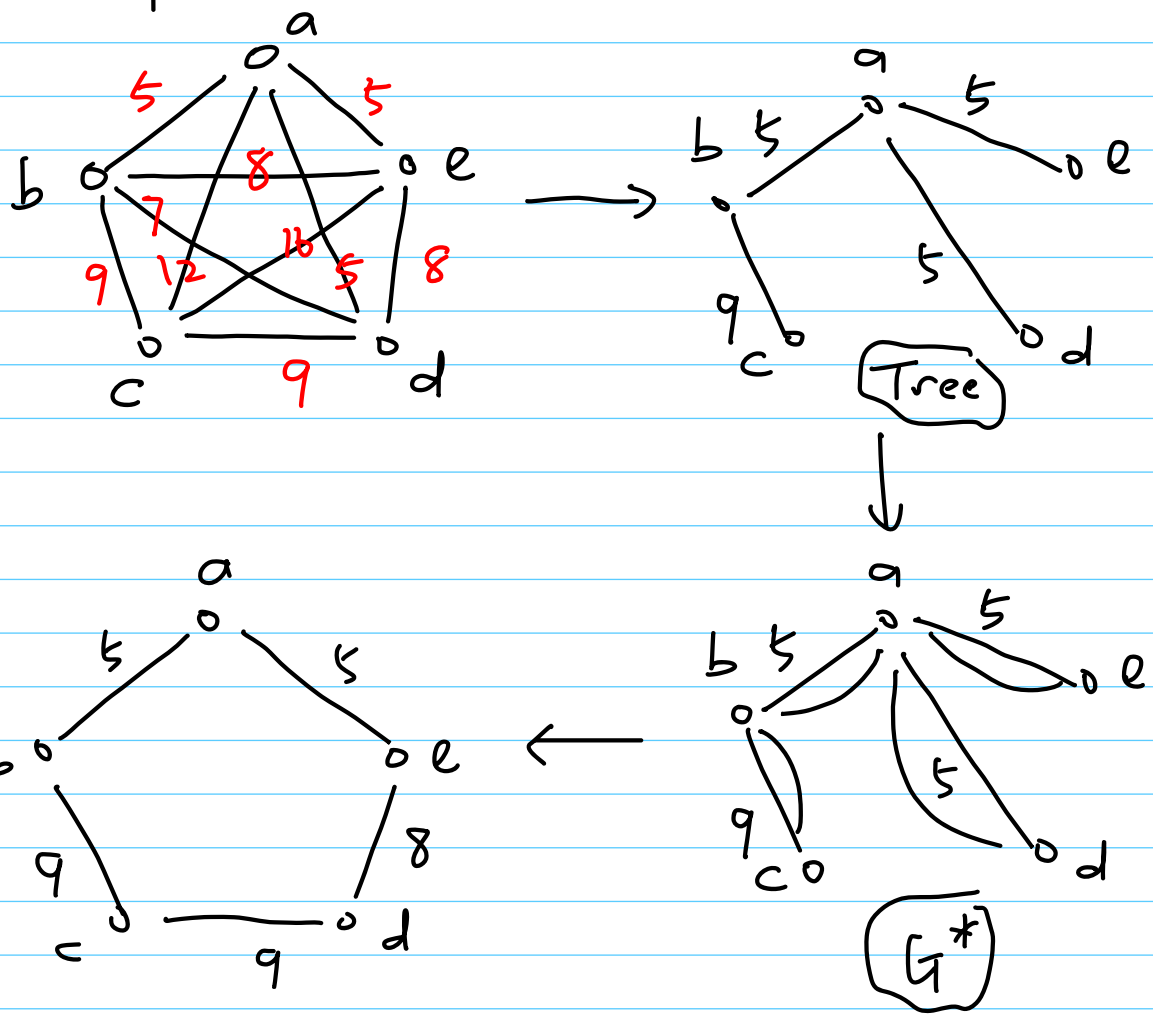


Theorem:

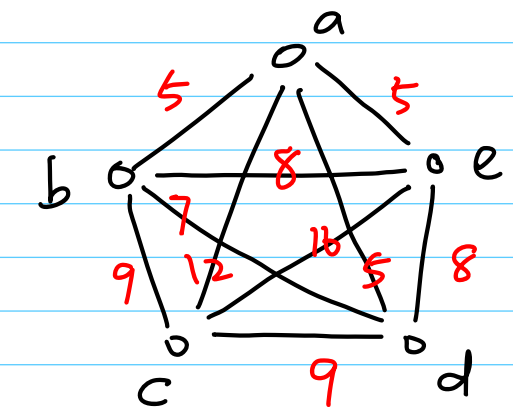
If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$, then G is hamiltonian.

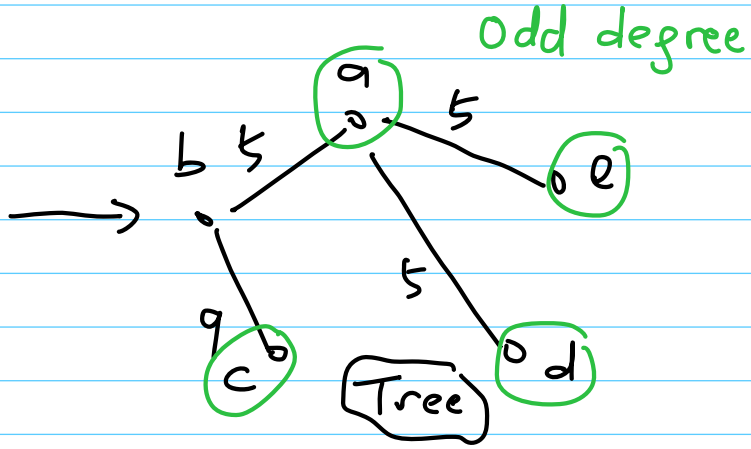
Traveling Salesman Problem

Example: <1>

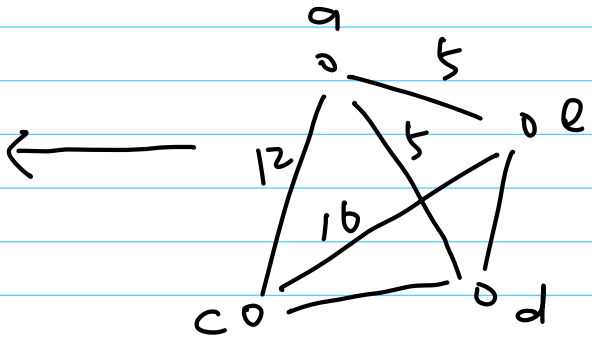


<2>

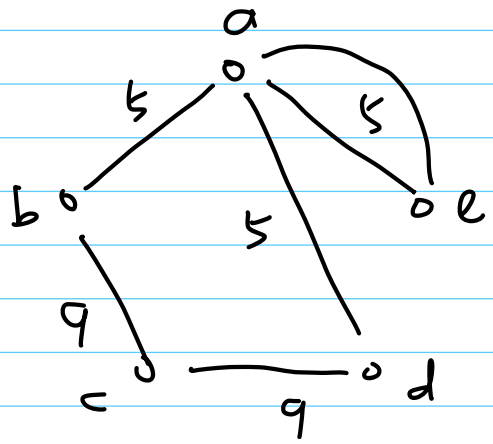

a
b
e
c
d
5
5
8
7
9
12
16
5
8
9


Odd degree
a
b
5
5
e
5
c
Tree
d

$V' = \{a, c, d, e\}$


a
5
e
12
5
16
c
d

$G[V']$


a
5
5
b
e
5
9
c
9
d